SIS Model for an Infectious Disease

The SIS model is a simple epidemilogical modeling for the transmission of an infectious disease without significant morbidity, such as the common cold or influenza, for which victims do not exhibit long-term immunity.

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SIS compartment model

An isolated and fixed population of N individuals is divided into two components. The first is a population of I individuals infected with a contagious disease, and the other comprises the $S = N - I$ individuals who are susceptible to the disease.

```
 _______________   alpha*I   _______________
|               |<---------|               |
|  Susceptible  |          |    Infected   |
|_______________|--------->|_______________|
                  beta*S*I
```

In any given period (we'll use a week as our basic unit of time), we assume a fraction α of the infected individuals will recover and return to the susceptible state. So the number of infected indivduals returning to the susceptible state is αI.

During that same period, each infected individual will encounter other individuals. The probability that an individual is susceptible is S/N, and the probability of infecting a susceptible individual will be denoted by β. Thus the average number of susceptible individuals becoming infected during the time period is $\beta S I$.

Writing this as a pair of differential equations for S and I, we find

$$\frac{dS}{dt} = \alpha I - \beta S I$$

$$\frac{dI}{dt} = -\alpha I + \beta S I$$

This can be simplified because $N = S + I$. Solving for S and substituting into the second equaton gives

$$\frac{dI}{dt} = -\alpha I + \beta (N - I) I$$

This is a single equation to be solved for I.

Paramter Values

```
N = 20000;      % Population size
alpha = 1;      % Fraction of infected individuals recovering in one period
beta = 5/N;     % Fraction of susceptible people infected by one person
```

Anonymous function for the Differential Equation

```
dI = @(t,I) - alpha*I + beta*I*(N-I);
```

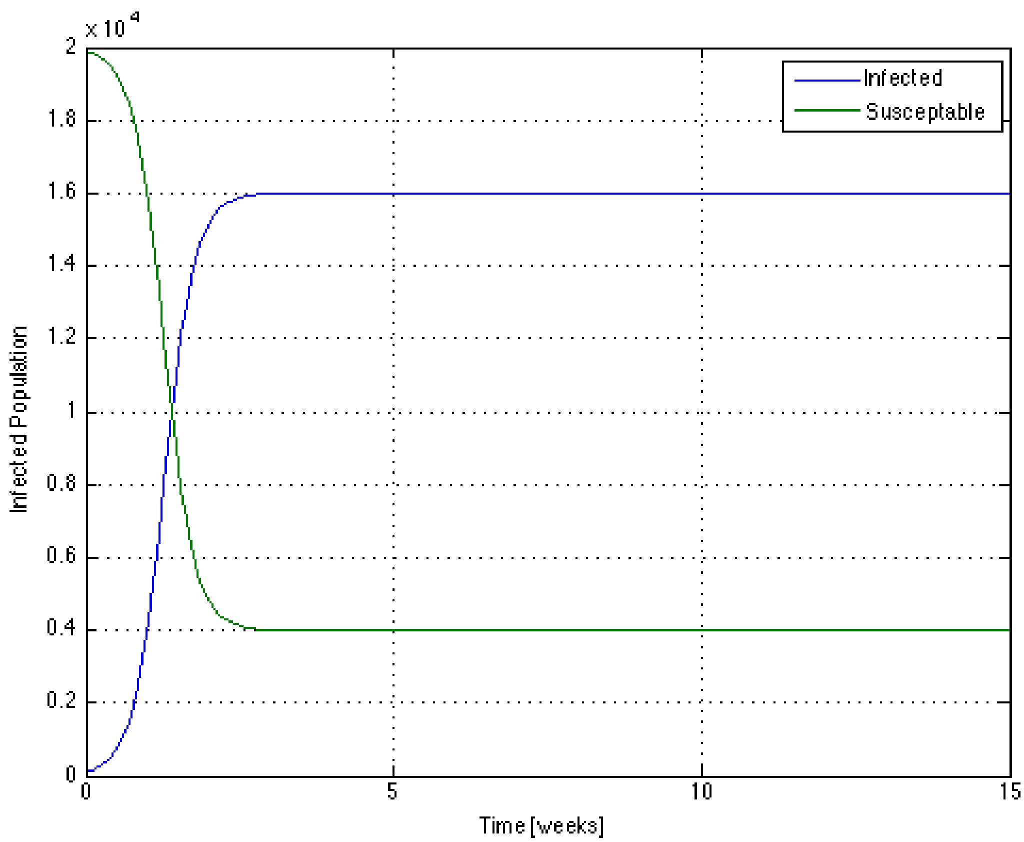
Solving the Differential Equation

```
I_initial = 100;    % Number of infected individuals at t_initial
t_initial = 0;      % Initial time
t_final = 15;       % Final time

[t,I] = ode45(dIdt,[t_initial t_final],I_initial);
```

Plotting the Result

```
plot(t,I,t,N-I);
grid;
xlabel('Time [weeks]');
ylabel('Infected Population');
legend('Infected','Susceptable');
```



Exercises

1. Repeat the simulation with different values for the initially infected population. What happens if $I = 0$? If $I = 1$? If $I = N$?

2. Public health efforts can reduce the probability of infection β. Use simulation to determine a target value for β. Is it possible to reduce the number of infected individuals be reduced to zero?

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