Control of the Ball and Beam System

File: `Ch08_BallBeam.m`

Contents

The ball and beam system is modeled using a second order differential equation

$$J\frac{d^2x}{dt^2} = -u$$

and associated control law. The Matlab function `ode45` integrates systems of first-order differential equations. So for simulation purposes, the first step is to recast the ball and beam system as a pair of first-order differential equations.

$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = -\frac{1}{J}u$$

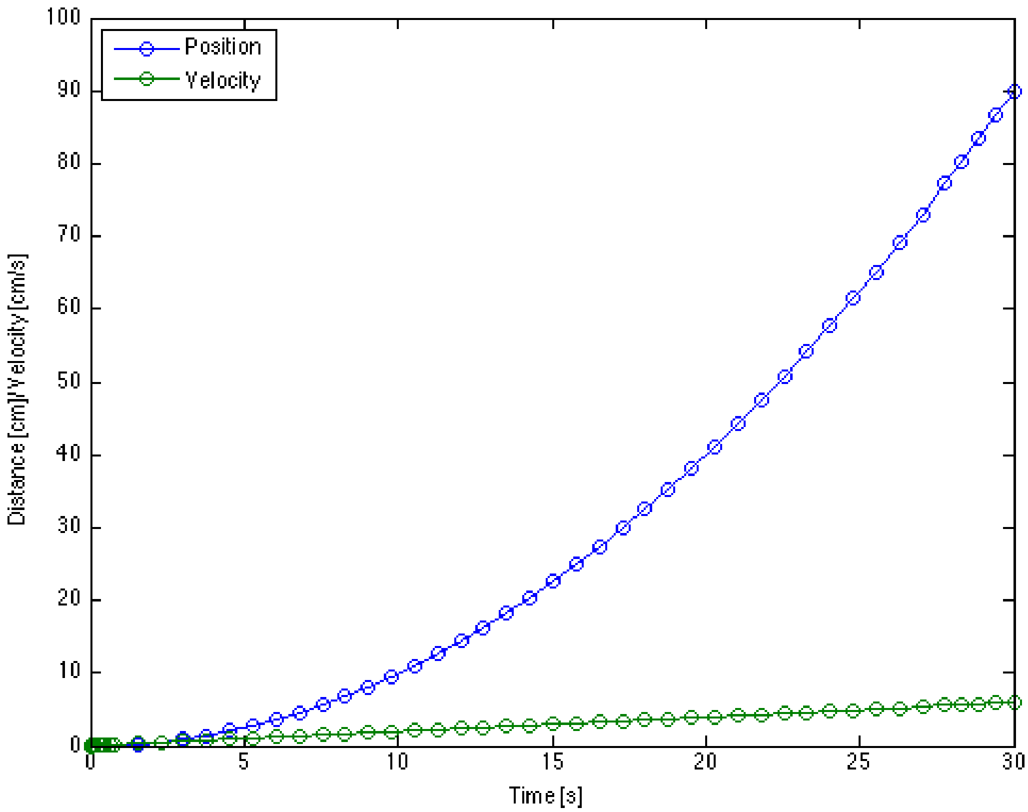
A Matlab vector must hold both x and v. We'll call this vector y such that $y_1 = x$ and $y_2 = v$. With this notation we have

$$\frac{dy_1}{dt} = y_2$$

$$\frac{dy_2}{dt} = -\frac{1}{J}u$$

The following Matlab code demonstrates this technique assume u has a constant value.

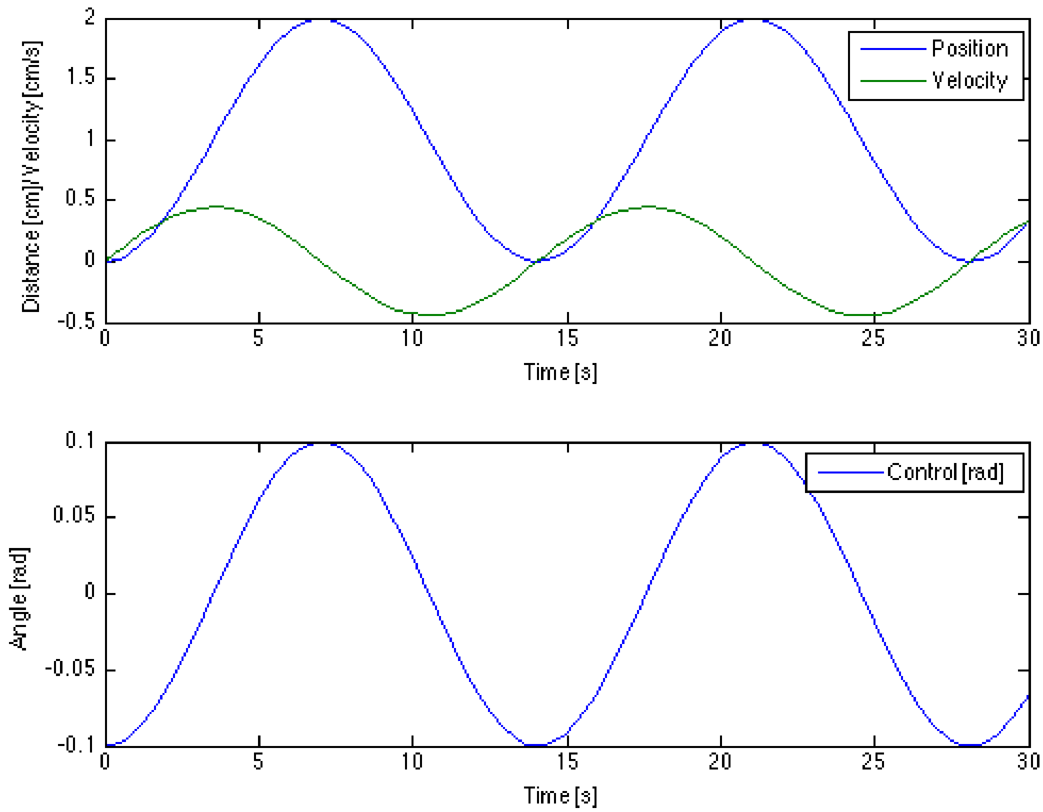
```
% Parameters
J = 0.5;          % Inertia
tspan = [0,30];   % Simulation time span
ic    = [0;0];    % Initial conditions

% Assume beam angle is a constant function
u = @(t,y) -0.1;

% Model Equations
f = @(t,y) [y(2); -u(t,y)/J];

% Integration and Plotting
ode45(f,tspan,ic);
legend('Position','Velocity','Location','NW');
xlabel('Time [s]');
ylabel('Distance [cm]/Velocity [cm/s]');
```



Proportional Control

Feedback control can be used to adjust the beam position in order to bring the ball to a desired position called the setpoint. Here we explore whether or not proportional control can do the job.

```
Kp  =  0.1;
xsp =  1;

% Proportional feedback control
u = @(t,y) Kp*(y(1)-xsp);

% Model Equations
f = @(t,y) [y(2); -u(t,y)/J];

% Integration and Plotting
[t,y] = ode45(f,tspan,ic);

subplot(2,1,1)
plot(t,y);
legend('Position','Velocity');
xlabel('Time [s]');
ylabel('Distance [cm]/Velocity [cm/s]');

subplot(2,1,2);
plot(t,arrayfun(@(t,x,v)u(t,[x;v]),t,y(:,1),y(:,2)));
legend('Control [rad]');
xlabel('Time [s]');
ylabel('Angle [rad]');
```

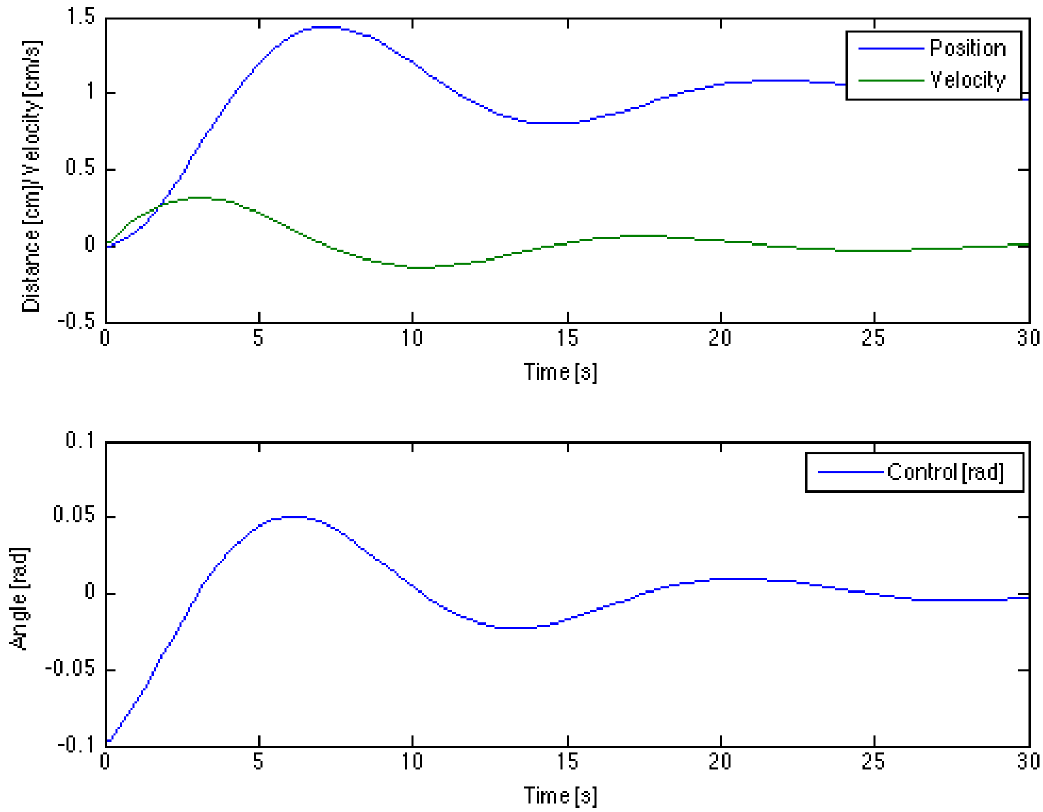


Proportional-Derivative Control (Underdamped)

Derviative action is needed to dampen the feedback control response for the ball and beam system. An analysis shows that the response to P-D control can be underdamped, overdamped, or critically damped.

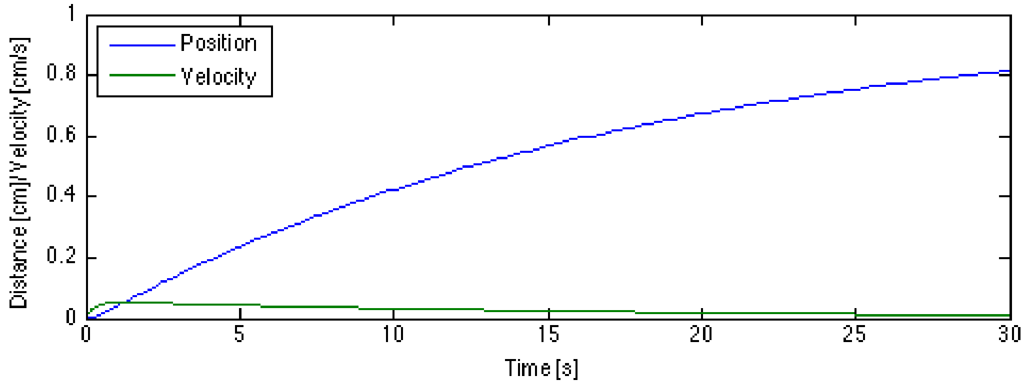
```
% Set Kd to a value 1/4 of that needed for critical damping.
Kd = 0.25*(sqrt(4*J*Kp));

% Proportional-Derivative Control
u = @(t,y) Kp*(y(1)-xsp) + Kd*y(2);

% Model Equations
f = @(t,y) [y(2); -u(t,y)/J];

% Integration and Plotting
[t,y] = ode45(f,tspan,ic);

subplot(2,1,1)
plot(t,y);
legend('Position','Velocity','Pred.');
xlabel('Time [s]');
ylabel('Distance [cm]/Velocity [cm/s]');

subplot(2,1,2);
plot(t,arrayfun(@(t,x,v)u(t,[x;v]),t,y(:,1),y(:,2)));
legend('Control [rad]');
xlabel('Time [s]');
ylabel('Angle [rad]');
```

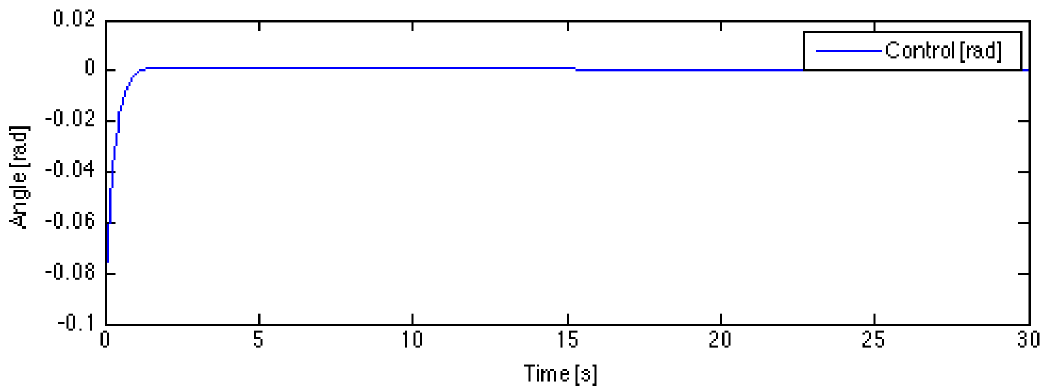
```
Warning: Ignoring extra legend entries.
```



Proportional-Derivative Control (Overdamped)

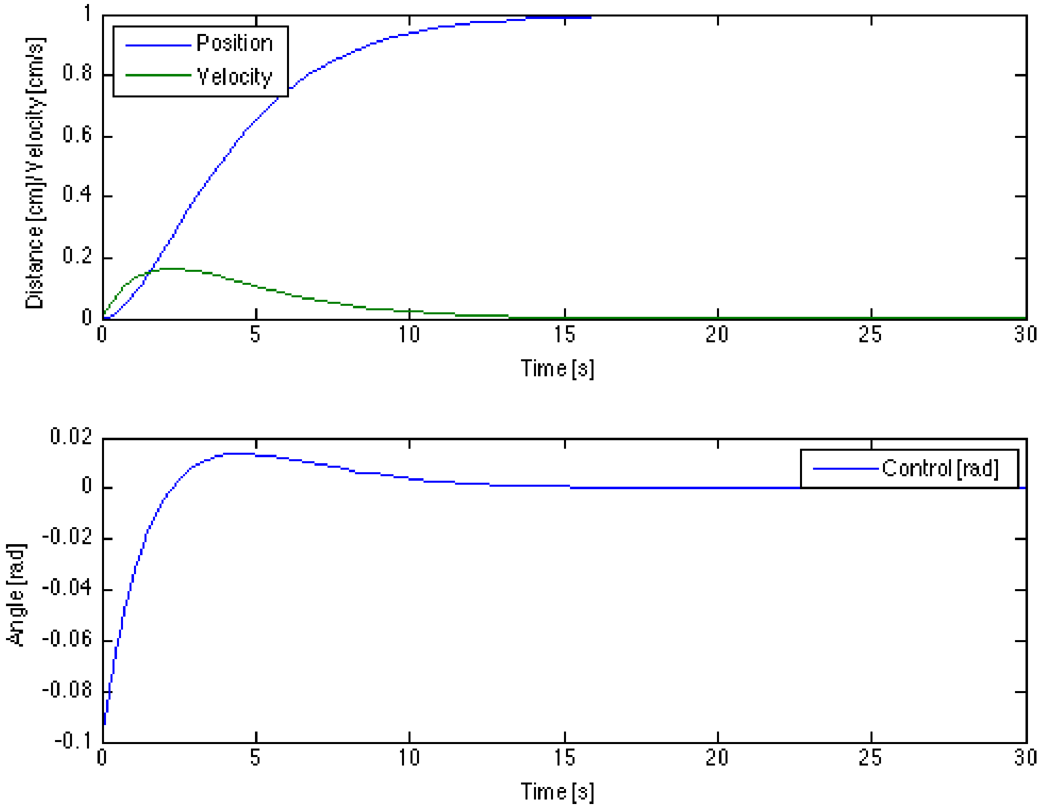
```
% Set Kd to a value 4 times of that needed for critical damping.
Kd = 4*(sqrt(4*J*Kp));

% Proportional-Derivative Control
u = @(t,y) Kp*(y(1)-xsp) + Kd*y(2);

% Model Equations
f = @(t,y) [y(2); -u(t,y)/J];

% Integration and Plotting
[t,y] = ode45(f,tspan,ic);

subplot(2,1,1)
plot(t,y);
legend('Position','Velocity','Location','NW');
xlabel('Time [s]');
ylabel('Distance [cm]/Velocity [cm/s]');

subplot(2,1,2);
plot(t,arrayfun(@(t,x,v)u(t,[x;v]),t,y(:,1),y(:,2)));
legend('Control [rad]');
xlabel('Time [s]');
ylabel('Angle [rad]');
```





Proportional-Derivative Control (Critically Damped)

```
% Set Kd to that needed for critical damping.
Kd = (sqrt(4*J*Kp));

% Proportional-Derivative Control
u = @(t,y) Kp*(y(1)-xsp) + Kd*y(2);

% Model Equations
f = @(t,y) [y(2); -u(t,y)/J];

% Integration and Plotting
[t,y] = ode45(f,tspan,ic);

subplot(2,1,1)
plot(t,y);
legend('Position','Velocity','Location','NW');
xlabel('Time [s]');
ylabel('Distance [cm]/Velocity [cm/s]');

subplot(2,1,2);
plot(t,arrayfun(@(t,x,v)u(t,[x;v]),t,y(:,1),y(:,2)));
legend('Control [rad]');
xlabel('Time [s]');
ylabel('Angle [rad]');
```



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