PID Control

File: `Ch12_PID.m`

Topics:

```
* PID Control
* Tuning Rules
```

To use the publish function with these notes, be sure you have the `displaytable.m` from the CBE30338 Utilities folder. Also, please note these notes use the Control Systems Toolbox, and require a reasonably current version of Matlab.clear all

Contents

SEMD Example 11.4 with Time Delay

The following diagram shows the basic elements of a feedback control system. The notation follows from Figure 11.8 of the SEMD textbook.

```
                                           _______
                             Disturbance  |       |
                                 d(s) --->| Gd(s) |-----
                                          |_______|     |
                  ______      _______      _______      v
Setpoint + _     |      |    |       |    |       |     _ +     Output
  r(s) -->|_|--->| K(s) |--->| Gv(s) |--->| Gp(s) |--->|_|-----> y(s)
           ^ -   |______|    |_______|    |_______|  +      |
           |    Controller Valve/Actuator  Process          |
           |                  _______                       |
           |                 |       |                      |
            -----------------| Gm(s) |----------------------
                             |_______|
                            Measurement
```

Transfer functions

Process

```
Gp = tf([1],[5 1],'TimeUnit','minutes')
```

```
Gp =

      1
  -------
  5 s + 1

Continuous-time transfer function.
```

Disturbance

```
Gd = Gp
```

```
Gd =

      1
  -------
  5 s + 1

Continuous-time transfer function.
```

Valve Actuator

```
Gv = tf([1],[2 1],'TimeUnit','minutes')
```

```
Gv =

      1
  -------
  2 s + 1

Continuous-time transfer function.
```

Measurement with Time Delay

```
Gm = tf([1],[1 1],'ioDelay',1,'TimeUnit','minutes')
```

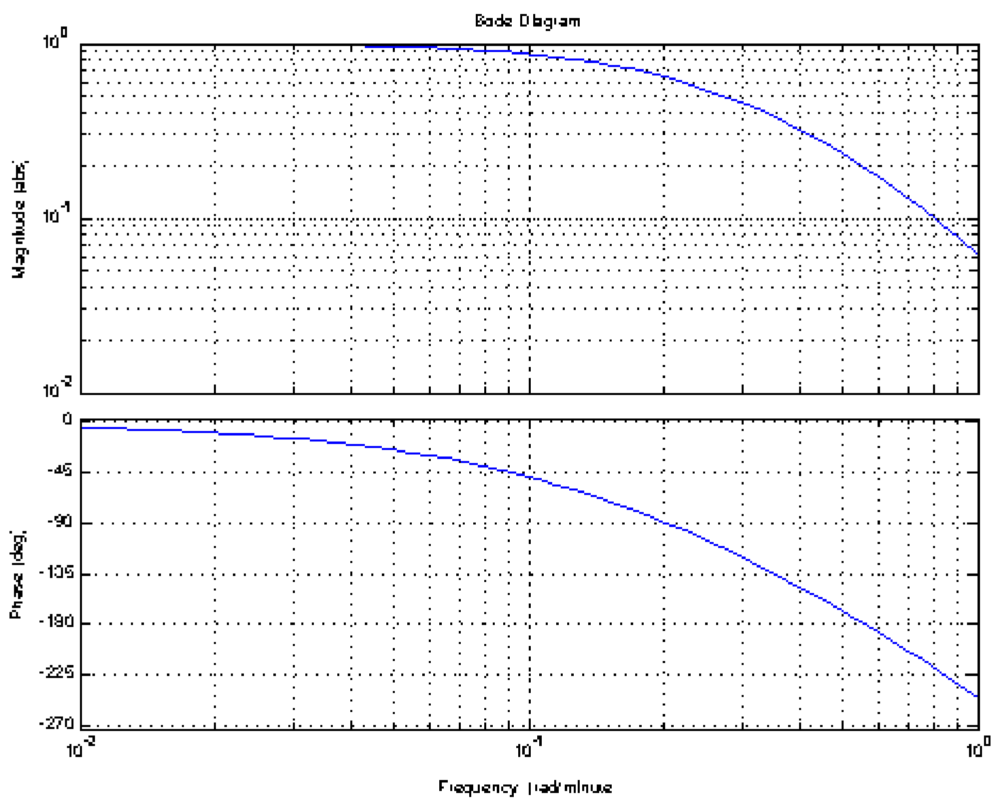
```
Gm =

                 1
  exp(-1*s) * -----
              s + 1

Continuous-time transfer function.
```

Gain Margin

Given the product of transfer functions Gm*Gp*Gv, the **gain margin** is the critical value of Kp for which the closed-loop becomes unstable. That critical value is also called the 'ultimate gain' Kcu.

The gain margin can be found from the Bode plot for Gm*Gp*Gv.

```
p = bodeoptions;
p.FreqUnits = 'rad/minute';
p.MagUnits = 'abs';
p.MagScale = 'log';

w = logspace(-2,0);

bodeplot(Gm*Gp*Gv,w,p);
grid;
```



Exercise: Use the Bode plot to estimate the cross-over frequency and Kcu.

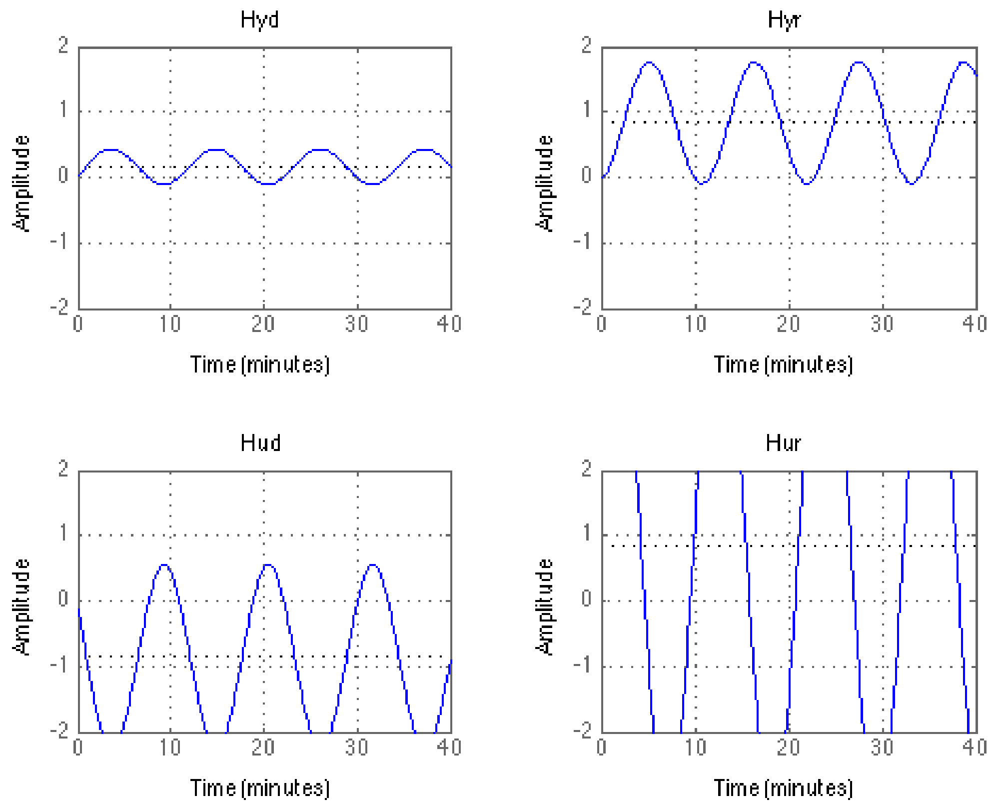
Ultimate Gain and Period

- Kcu, the **ultimate control gain**, is the value proportional for which the closed loop system exhibits a sustained oscillation. This is equal in value to the gain margin.
- Pu, the **ultimate period**, is the period of sustained oscillation when the proportional control gain is Kcu.

- he gain margin and the cross-over frequency can be computed with the Matlab function `margin`.

```
[gmargin,~,wco] = margin(Gm*Gp*Gv);
displaytable([gmargin;wco],{'Gain Margin';'Crossover Freq [rad/min]'});

Kcu = gmargin;
Pu = 2*pi/wco;
displaytable([Kcu;Pu],{'Kcu';'Pu'});

% Closed-loop transfer function at the Ultimate Gain

K = tf([Kcu],[1],'TimeUnit','minutes');

Hyd =      Gd/(1 + Gp*Gv*K*Gm);
Hyr = Gp*Gv*K/(1 + Gp*Gv*K*Gm);
Hud =   -K*Gd/(1 + K*Gm*Gp*Gv);
Hur =       K/(1 + K*Gm*Gp*Gv);

% Plot Step Responses

t = 0:0.1:40;
ax = [min(t) max(t) -2 2];
subplot(2,2,1);step(Hyd,t);axis(ax);grid;title('Hyd');
subplot(2,2,2);step(Hyr,t);axis(ax);grid;title('Hyr');
subplot(2,2,3);step(Hud,t);axis(ax);grid;title('Hud');
subplot(2,2,4);step(Hur,t);axis(ax);grid;title('Hur');
```

```
Gain Margin                5.1215
Crossover Freq [rad/min]  0.56032

Kcu   5.1215
Pu    11.213
```



Observations

- Marginal stability. Try increasing and decreasing Kp to see what happens.
- Period of Oscillation corresponds to the cross over frequency.

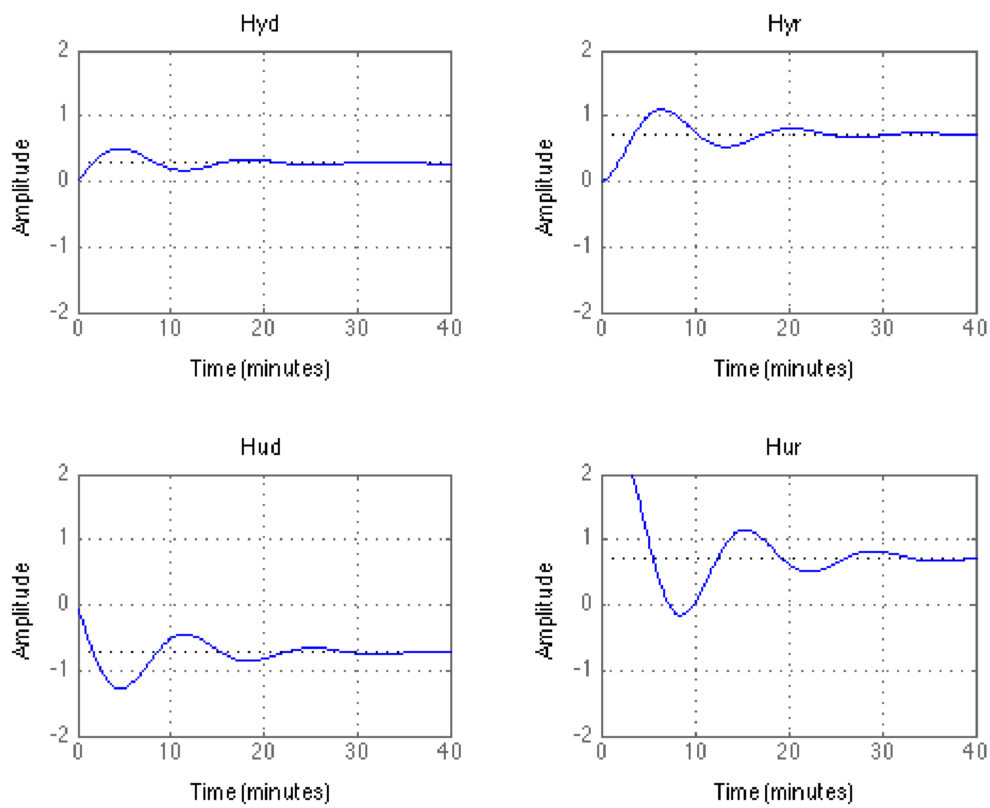
Ziegler-Nichols Tuning Rule: P

The Ziegler-Nichols tuning rules are shown in Table 12.4 on page 224 of the SEMD textbook. The proportional-only control, the control gain is set to 1/2 of the ultimate gain determined by experiment or from the Bode polot.

```
Kp = 0.5*Kcu;
displaytable(Kp,'Kp');

K = tf([Kp],[1],'TimeUnit','minutes');

% Closed-loop transfer functions

Hyd =       Gd/(1 + Gp*Gv*K*Gm);
Hyr = Gp*Gv*K/(1 + Gp*Gv*K*Gm);
Hud =    -K*Gd/(1 + K*Gm*Gp*Gv);
Hur =        K/(1 + K*Gm*Gp*Gv);

% Plot Step Responses

t = 0:0.1:40;
ax = [min(t) max(t) -2 2];
subplot(2,2,1);step(Hyd,t);axis(ax);grid;title('Hyd');
subplot(2,2,2);step(Hyr,t);axis(ax);grid;title('Hyr');
```

```
subplot(2,2,3);step(Hud,t);axis(ax);grid;title('Hud');
subplot(2,2,4);step(Hur,t);axis(ax);grid;title('Hur');
```

```
Kp    2.5607
```



There are several problems with this proportional-only controller

- The step responses are underdamped
- Steady state offset is evident in the disturbance and setpoint responses.
- Significant control action is required for the setpoint response.

With proportional-only control there is an unfortunate tradeoff between damping and offset.

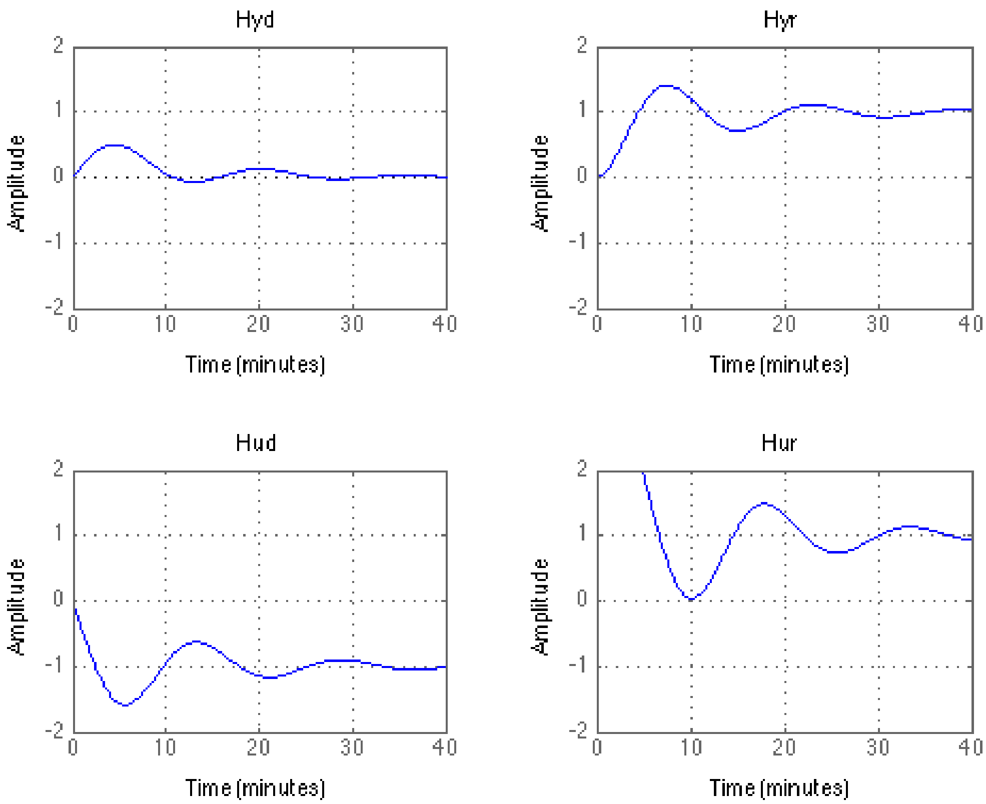
Ziegler-Nichols Tuning Rule: PI

Integral control eliminates offset. Ziegler-Nichols tuning rule (Table 12.4, page 224)

```
% Ultimate gain, crossover frequency, and ultimate period

[Kcu,~,wco] = margin(Gm*Gp*Gv);
Pu = 2*pi/wco;

% Ziegler-Nichols PI Tuning Rules
```

```
Kp = 0.45*Kcu;
Ti = Pu/1.2;

displaytable([Kp;Ti],{'Kp';'Ti'});

% PI Controller

P = Kp*tf([1],[1],'TimeUnit','minutes');
I = Kp*tf([1],[Ti 0],'TimeUnit','minutes');

K = P + I;

% Closed-loop transfer functions

Hyd =      Gd/(1 + Gp*Gv*K*Gm);
Hyr = Gp*Gv*K/(1 + Gp*Gv*K*Gm);
Hud =   -K*Gd/(1 + K*Gm*Gp*Gv);
Hur =       K/(1 + K*Gm*Gp*Gv);

% Plot Step Responses

t = 0:0.1:40;
ax = [min(t) max(t) -2 2];
subplot(2,2,1);step(Hyd,t);axis(ax);grid;title('Hyd');
subplot(2,2,2);step(Hyr,t);axis(ax);grid;title('Hyr');
subplot(2,2,3);step(Hud,t);axis(ax);grid;title('Hud');
subplot(2,2,4);step(Hur,t);axis(ax);grid;title('Hur');
```

```
Kp  2.3047
Ti  9.3446
```



Observations

- Steady state offset is gone (due to integral action).
- Step responses are still underdamped.
- Significant control action is required for the setpoint response.

Take time to do a careful comparison. Try changing the control parameters to see what happens when you increase and decrease the integral time constant.

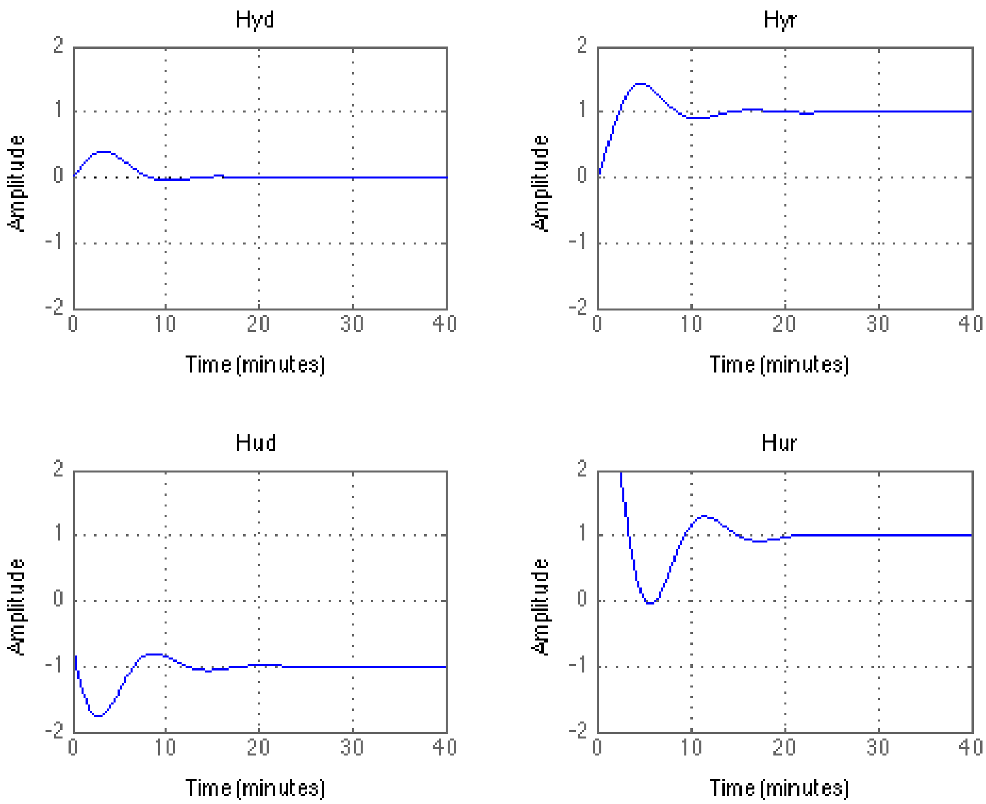
Ziegler-Nichols Tuning Rule: PID

Derivative action is used to increase damping. The increased damping also allows somewhat larger proportional control gains and shorter integral time constants. Ziegler-Nichols tuning rule (Table 12.4, page 224)

```
% Ultimate gain, crossover frequency, and ultimate period

[Kcu,~,wco] = margin(Gm*Gp*Gv);
Pu = 2*pi/wco;

% Ziegler-Nichols PID Tuning Rules

Kp = 0.6*Kcu;
Ti = Pu/2;
Td = Pu/8.0;
N= 10;

displaytable([Kp;Ti;Td;N],{'Kp';'Ti';'Td';'N'});
```

```
% PID Controller

P = Kp*tf([1],[1],'TimeUnit','minutes');
I = Kp*tf([1],[Ti 0],'TimeUnit','minutes');
D = Kp*tf([Td 0],[Td/N 1],'TimeUnit','minutes');

K = P + I + D;

% Closed-loop transfer functions

Hyd =      Gd/(1 + Gp*Gv*K*Gm);
Hyr = Gp*Gv*K/(1 + Gp*Gv*K*Gm);
Hud =   -K*Gd/(1 + K*Gm*Gp*Gv);
Hur =       K/(1 + K*Gm*Gp*Gv);

% Plot Step Responses

t = 0:0.1:40;
ax = [min(t) max(t) -2 2];
subplot(2,2,1);step(Hyd,t);axis(ax);grid;title('Hyd');
subplot(2,2,2);step(Hyr,t);axis(ax);grid;title('Hyr');
subplot(2,2,3);step(Hud,t);axis(ax);grid;title('Hud');
subplot(2,2,4);step(Hur,t);axis(ax);grid;title('Hur');
```

```
Kp  3.0729
Ti  5.6067
Td  1.4017
N       10
```



Observations

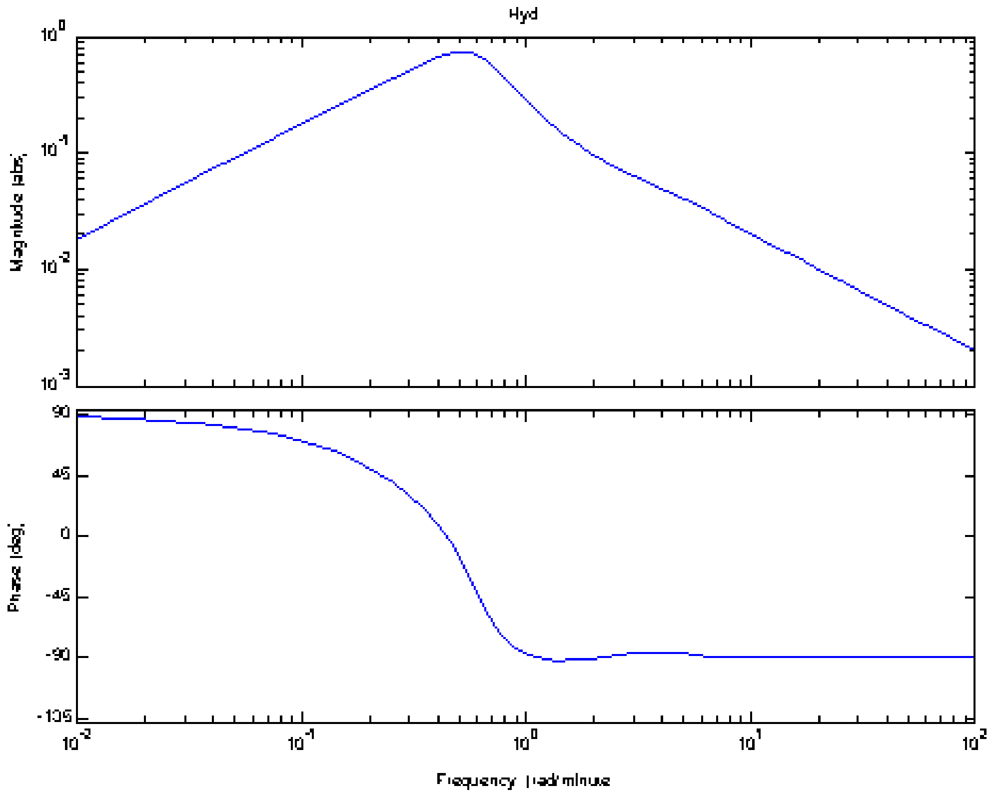
- Better tracking to steady state
- Better damping
- Significant control action is still required for the setpoint response.

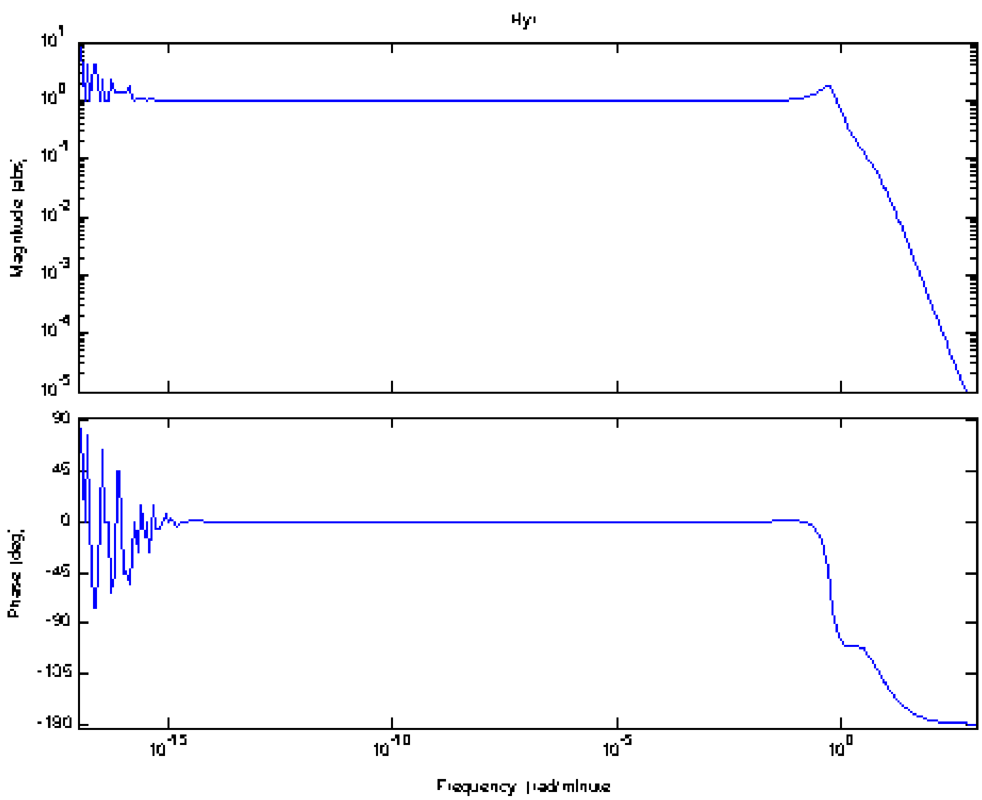
Take time to do a careful comparison. Try changing the control parameters to see what happens when you increase and decrease the control parameters.

Closed Loop Transfer functions

Examine Bode plots for the closed-loop transfer functions. Can you see the relationships between these plots and the observed step responses?

```
figure(1);clf;
bodeplot(Hyd,p);
title('Hyd');

figure(2);clf;
bodeplot(Hyr,p);
title('Hyr');
```


Hyd
Magnitude (abs)
Phase (deg)
Frequency (rad/minute)


Hyr
Magnitude (abs)
Phase (deg)
Frequency (rad/minute)