

PID Two Degrees of Freedom

File: Ch12_PID_2DOF.m

Topics:

- * PID Control
- * Tuning Rules

To use the publish function with these notes, be sure you have the displaytable.m from the CBE30338 Utilities folder. Also, please note these notes use the Control Systems Toolbox, and require a reasonably current version of Matlab.

Contents

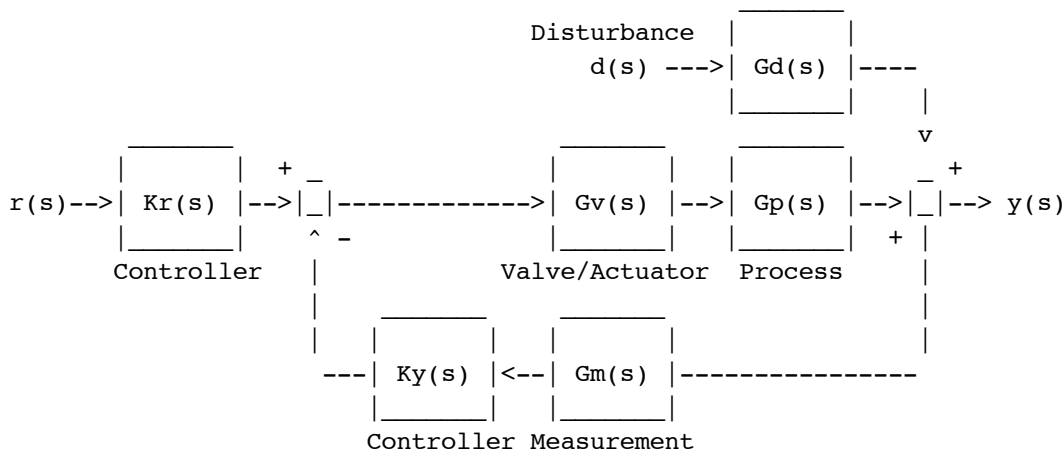
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SEMD Example 11.4 with Time Delay

The following diagram shows the basic elements of a feedback control system. D

Two Degree of Freedom Controller

Further improvement in the closed-loop response is possible reconfigure the control loop to provide independent control of the disturbance rejection and setpoint response properties.



Transfer functions

Process

```
Gp = tf([1],[5 1], 'TimeUnit', 'minutes')
```

Gp =

$$\frac{1}{5s + 1}$$

Continuous-time transfer function.

Disturbance

```
Gd = Gp
```

Gd =

$$\frac{1}{5s + 1}$$

Continuous-time transfer function.

Valve Actuator

```
Gv = tf([1],[2 1], 'TimeUnit', 'minutes')
```

Gv =

$$\frac{1}{2s + 1}$$

Continuous-time transfer function.

Measurement with Time Delay

```
Gm = tf([1],[1 1], 'ioDelay', 1, 'TimeUnit', 'minutes')
```

Gm =

$$\exp(-1*s) * \frac{1}{s + 1}$$

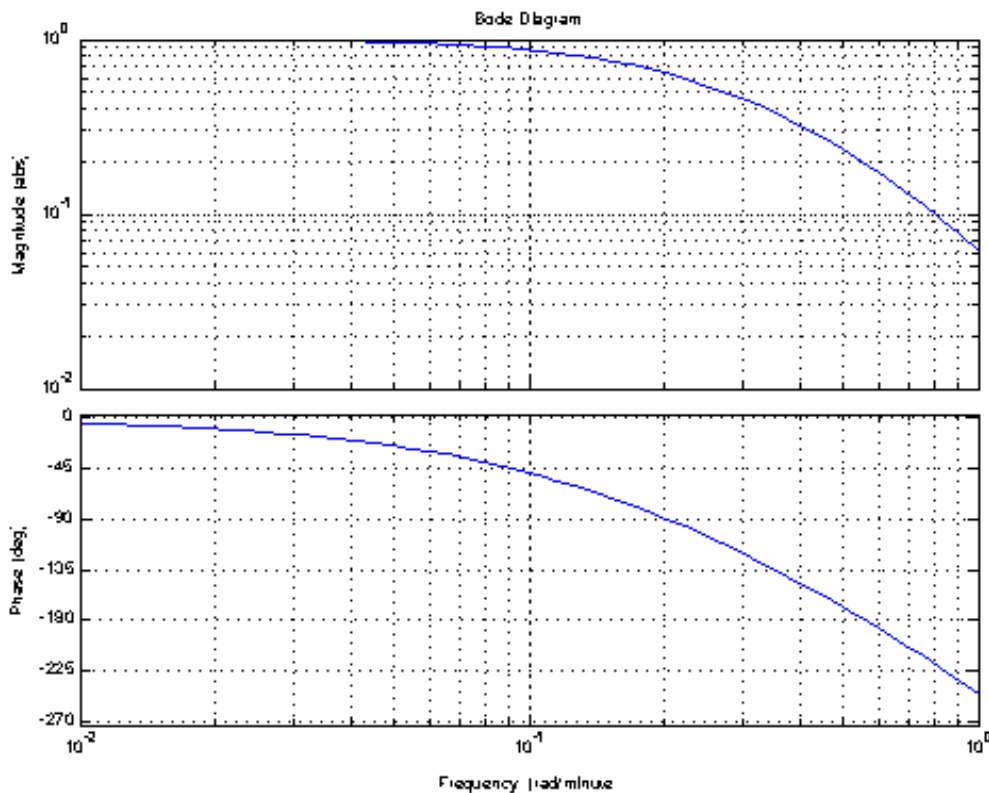
Continuous-time transfer function.

Gain Margin

Given the product of transfer functions $G_m*G_p*G_v$, the **gain margin** is the critical value of K_p for which the closed-loop becomes unstable. That critical value is called the 'ultimate gain' K_{cu} .

The gain margin can be found from the Bode plot for $G_m*G_p*G_v$.

```
p = bodeoptions;  
p.FreqUnits = 'rad/minute';  
p.MagUnits = 'abs';  
p.MagScale = 'log';  
  
w = logspace(-2,0);  
  
bodeplot(Gm*Gp*Gv,w,p);  
grid;
```



Before going further, use the Bode plot to estimate the cross-over frequency and K_{cu} .

Two Degree of Freedom Controller

The maximum proportional gain is the gain margin. The gain margin and the cross-over frequency can be computed with the Matlab function `margin`.

```

addpath('utilities');

[Kcu,~,wco] = margin(Gm*Gp*Gv);
displaytable([Kcu;wco],{'Gain Margin';'Crossover Freq [rad/min]'});

Pu = 2*pi/wco;

P = @(Kp) tf([Kp],[1],'TimeUnit','minutes');
I = @(Ti) tf([1],[Ti 0],'TimeUnit','minutes');
D = @(Td) tf([Td 0],[Td/10 1],'TimeUnit','minutes');

PID = @(Kp,Ti,Td,beta,gamma) P(Kp)*(beta + I(Ti) + gamma*D(Td));

% Closed-loop transfer functions

Hyd = @(Ky,Kr) Gd/(1 + Gp*Gv*Ky*Gm);
Hyr = @(Ky,Kr) Gp*Gv*Kr/(1 + Gp*Gv*Ky*Gm);
Hud = @(Ky,Kr) -Ky*Gd/(1 + Ky*Gm*Gp*Gv);
Hur = @(Ky,Kr) Kr/(1 + Ky*Gm*Gp*Gv);

% Ziegler Nichols

Kzn = PID(0.6*Kcu,Pu/2,Pu/8,1,1);

% Plot Step Responses

t = 0:0.1:40;
ax = [min(t) max(t) -2 2];

subplot(7,2,[1 3 5]);
hHyd = plot(t,step(Hyd(Kzn,Kzn),t));
hold on; plot(t,0*sign(t),'r--'); hold off;
ylabel('y(t): Output');legend('Hyd');axis(ax);grid;
title('Disturbance Step Response');

subplot(7,2,[2 4 6]);
hHyr = plot(t,step(Hyr(Kzn,Kzn),t));
hold on; plot(t,sign(t),'r--'); hold off;
ylabel('y(t): Output');legend('Hyr');axis(ax);grid;
title('Setpoint Step Response');

subplot(7,2,[7 9 11]);
hHud = plot(t,step(Hud(Kzn,Kzn),t));
ylabel('u(t): Control');legend('Hud');axis(ax);grid;

subplot(7,2,[8 10 12]);
hHur = plot(t,step(Hur(Kzn,Kzn),t));
ylabel('u(t): Control');legend('Hur');axis(ax);grid;

update = @(Kp,Ti,Td,beta,gamma) {
    suiPlot(hHyd,t,step(Hyd(PID(Kp,Ti,Td,1,1),PID(Kp,Ti,Td,beta,gamma)),t));
    suiPlot(hHyr,t,step(Hyr(PID(Kp,Ti,Td,1,1),PID(Kp,Ti,Td,beta,gamma)),t));
    suiPlot(hHud,t,step(Hud(PID(Kp,Ti,Td,1,1),PID(Kp,Ti,Td,beta,gamma)),t));
    suiPlot(hHur,t,step(Hur(PID(Kp,Ti,Td,1,1),PID(Kp,Ti,Td,beta,gamma)),t));
};

[Kp,loc] = suiSlider(0,Kcu,0.6*Kcu,'Kp');

```

```

[beta] = suiSlider(0,1,1,'beta',loc.Above);
[invTi,loc] = suiSlider(0.01,5/Pu,2/Pu,'1/Ti',loc.Right);
[Td,loc] = suiSlider(0,Pu,Pu/8,'Td',loc.Right);
[gamma] = suiSlider(0,1,1,'gamma',loc.Above);

zntuning = @() {
    Kp(0.6*Kcu);
    invTi(2/Pu);
    Td(Pu/8)};

suiButton(zntuning,'ZN',loc.Right);

suiUpdate(@()update(Kp(),1/invTi(),Td(),beta(),gamma()));

```

Gain Margin 5.1215
Crossover Freq [rad/min] 0.56032

